THE FOUR NUMBERS GAME, WEEK 3

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We came back to the question of why the powers of two give us games that terminate.

We tried changing the problem to a different one, where instead of taking the absolute differences, we added the numbers and then modded by 2. We found that if we got everything to be 0 through mod 2 addition, we would get even numbers by adding without taking mod 2. And if we got all even numbers through addition, we'd get even numbers through subtraction. And finally if we got all even numbers through subtraction, we'd get all even numbers through subtracting and then taking the absolute values.

In summary, if we could get all 0s through adding mod 2, we'd be able to get all even numbers through subtracting and taking absolute values.

As a special case, if we suppose that we can show that we get all 0s through adding mod 2, then if we have all 0s and 1s (in the case of absolute subtraction) we must eventually get all even numbers, which would mean we have all 0s.

Taking a different path, we tried to decompose the Eight-Numbers game into the Four-Numbers game. We noticed that if we had an Eight-Numbers game with two adjacent 1s and the rest 0s, we could embed a Four-Numbers game, with one vertex pointing between the 1s, with a one at that vertex and 0s in the other three corners. Then every two steps in the Eight-Numbers game corresponds to a single step in the embedded Four-Numbers game, with a 1 in the Four-Numbers game matching a pair of adjacent 1s in the Eight-Numbers game. If the embedded Four-Numbers game terminates, then the surrounding Eight-Numbers game has to terminate.

So we asked whether we could always embed a Four-Numbers game in any Eight-Numbers game, as well as why this didn't work for a Twelve-Numbers game.