THE FOUR NUMBERS GAME, WEEK 3

MATTHEW TAI

We took another look at embedding smaller games into larger games. After deciding that two adjacent 1s on an octagon could be translated into a single 1 on an embedded square, we tried to determine what to do in the case that there was only one 1, with the adjacent number being a 0.

Then we hit upon the idea of making the corresponding entry on the square $\frac{1}{2}$. Perhaps if we take the averages, we could fill in the inner square.

So we tried out several examples, and they seemed to work, so we tried to write it out generally.

We considered at four vertices next to each other, with values a, b, c, and d. We want the result of averaging to get to the square and then taking a step to be the same as taking two steps on the octagon and then averaging to get to the square. More explicitly, we considered the average of a and b, $\frac{a+b}{2}$, and the average of c and d, $\frac{c+d}{2}$, and considered their absolute difference,

$$|\frac{a+b}{2} - \frac{c+d}{2}|$$

Then we considered the absolute differences of a and b, b and c, and c and d, which were $|a-b|$, $|b-c|$, and $|c-d|$ respectively. Then we took the absolute differences of $|a-b|$ and $|b-c|$ and of $|b-c|$ and $|c-d|$, getting

$$||a-b|-|b-c||$$

and

$$||b-c|-|c-d||$$

Then we took the average, to get

$$\frac{||a-b|-|b-c||+||b-c|-|c-d||}{2}$$

So we wanted to show that

$$\frac{||a-b|-|b-c||+||b-c|-|c-d||}{2} = |\frac{a+b}{2} - \frac{c+d}{2}|$$

If we could show this, then we would have shown that any octagon has an embedded square and thus, since we've shown that the Four Numbers game always terminates, the Eight Numbers game would always terminate. Similarly, we would be able to embed an Eight Numbers game into a Sixteen Numbers game and so on.

But then we hit upon a disaster. If we take the four numbers a, b, c and d to be $1, 2, 3$, and 4, then we get

$$||1-2|-|2-3|| = |1-1| = 0$$

and

$$||2-3|-|3-4|| = |1-1| = 0$$

so the left hand side of the equation above would be 0, while we get

$$|\frac{1+2}{2} - \frac{3+4}{2}| = |\frac{3}{2} - \frac{7}{2}| = 2$$

They don't match up! So therefore we can't always use the embedded square to describe the outer octagon!

So we have to directions to go in: we could either try to find out why some sets of numbers don't work and thus try to remedy the method, or we could try to find something else.