MAKING THE REALS FROM THE RATIONALS, WEEK 3

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Still working toward a good notion of "arbitrarily close" we asked what happens if we have a sequence that converges to two different things, such as $(1, \frac{1}{2}, 1, \frac{1}{3}, 1...)$ where taking every other term gave us either a sequence that went to 1 or a sequence that went to 0.

This led to another idea, of what would happen if both sequences went to the same number. The sequence $(0, \frac{1}{2}, 0, \frac{1}{3}, 0, ...)$ can be split into two sequences that both go to 0. We decided that the composite sequence does in fact go to 0, despite successive terms not getting strictly closer to 0.

So I asked whether a sequence made up of three sequences that all go to 0 could be considered to go to 0. The consensus was that yes, the composite sequence also went to 0.

Then I asked about the sequence of mixed 0s and descending fractions where the number of 0s between fractions was random. Once again we got into the conversation of whether all the sequences had to have patterns or not.

As it turns out, the number of patterns is unfortunately countable. Any pattern that one can describe can be written down, and from there they can be alphabetized. All the one symbol patterns can be listed alphabetically, then all the two symbol patterns, and so on, so that we have a list of patterns in a one-to-one correspondence with the natural numbers.

But the real numbers were not countable; the number of real numbers is a larger infinity than the countable infinity.

At this point we reached the question of what does it mean for one infinity to be larger than another infinity. We considered an analogy of two boxes. One box is said to be smaller than another box if the first box can be placed into the second box but the second box cannot be put into the first box.

We can put the natural numbers into the real numbers, but as proven by Cantor's Diagonal argument, we can't put the real numbers into the natural numbers.

Just for completeness, we also discussed the larger infinities, the tower of $\aleph$s, as well as the infinities that could be generated from them. We used the metaphor of the Infinite Hotel, with $\aleph_0$ rooms that could fit one more person in, or another $\aleph_0$ people, but not $\aleph_1$, and so on.

Unfortunately, having all of these infinities didn't bring us to a good definition by the time class ended.