MAKING THE REALS FROM THE RATIONALS, WEEK 4

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We took another look at what it might mean for a sequence to tend toward something.

We looked at the sequence

$$(1, 1.4, 1.41, 1.414, 1.4142, ...)$$

which spells out the decimal digits of $\sqrt{2}$, or, alternatively, is the largest rational number with n decimal digits that squares to less than 2. Why does this sequence mean $\sqrt{2}$?

We hit upon the idea of saying that a sequence goes to the smallest number that it doesn't reach. We also proposed saying that a sequence goes to the smallest number that it doesn't get larger than. But these are slightly different, as shown by the sequence

$$(1, 1, 1, ...)$$

So we went with the second defintion.

But what about descending sequences, like

$$(1, \frac{1}{2}, \frac{1}{3}, ...)$$

The smallest number that it doesn't get bigger than is 1, but the sequence doesn't go to 1, it goes to 0. So in this case we say that it goes to the largest number it doesn't get smaller than.

So what about sequences that alternate getting higher and lower? Such as

$$(1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, ...)$$

Here we can't say that it goes to the lowest number that it doesn't get larger than, 1, or the highest number it doesn't get lower than, $-\frac{1}{2}$.

We also looked again at

$$(0, \frac{1}{2}, 0, \frac{1}{3}, ...)$$

to get an idea of how we could deal with a sequence that sometimes got farther away from the number it was supposedly tending toward. What if we said that a number that wasn't the target couldn't be in the sequence more than once? Or perhaps at least not twice in a row?

But what about the sequence

$$(0, \frac{1}{2}, 0, 1, 1, 1, 1, ...)$$

which goes to 1, but has several 0s in it? Or the sequence

$$(0, 0, 0, \frac{1}{2}, 0, 1, 1, 1, ...)$$

which has several 0s in a row?

Then we came to the conclusion that we could take off finitely many entries as we wanted off of the front, those 0s didn't matter. But how many 0s can we have in a sequence that ultimately goes to 1?

At the end of the class, we took another look at

$$(0, \frac{1}{2}, 0, \frac{1}{3}, ...)$$

and asked if after the $\frac{1}{2}$ the sequence ever got more than $\frac{1}{2}$ away from 0, using the distance measure $|a - b|$ to be the distance between a and b. The answer was no. I then asked if the sequence ever got more than $\frac{1}{2}$ away from $\frac{1}{2}$; once again, the answer was no.

So then I asked if after the $\frac{1}{3}$ the sequence ever got more than $\frac{1}{3}$ away from 0, and if it ever got more than $\frac{1}{3}$ away from $\frac{1}{3}$. Once again, the answer was no.

I told them to think about those last few statements to see if they could come up with anything.